Final Review, Part 2

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1 Lab takeaways

1.1 Radiometer lab

In this lab, you constructed some simple circuits to demonstrate the effects of the radiometer and Friis equations. For the Friis equation, you demonstrated that the low-noise component should go first to minimize the overall noise temperature of the circuit; the components with larger noise temperatures will have those temperatures "damped" by the gains of the components before them. For the radiometer equation, you used some basic statistics to compute ΔT for a radiometer. You observed that, in general, the trend followed $\Delta T \propto \tau^{-1/2}$, where τ is the integration time. However, you also observed a "knee" feature at which the radiometer equation ceased to apply. From the NRAO radio astronomy course:

> Gain fluctuations typically have a "$1/f$" power spectrum, where f is the post-detection frequency, so they are larger on longer time scales and increasing τ eventually results in a higher noise level. The gain stability of a receiver is often specified by the "$1/f$ knee" f_{knee}, the postdetection frequency at which $\sigma_{\text{noise}} = \sigma_G$. Integrations longer than $\tau \approx 1/(2\pi f_{\text{knee}})$ will likely increase the noise level.

1.2 CMB detection lab

In this lab, you endeavored to measure the temperature of the cosmic microwave background. This lab was largely focused on methods and sources of error, rather than the actual origin or theory of the CMB. First, you performed a beam mapping exercise, where a "loud" radio source was placed near the telescope and the telescope was tilted on its axis to determine the gain as a function of angle. Next, you did tipping scans to determine the contribution of the atmosphere to your measurements. You also did a hot-cold load calibration, where "hot" (room temperature) and "cold" (liquid nitrogen) loads were placed directly in front of the telescope and the total power was measured. This can be used (with or without the intermediate Y-factor) to compute the receiver temperature.

1.3 Tabletop interferometry lab

In this lab, you used an *adding* interferometer and compact fluorescent lights (CFLs) to simulate sources on the "sky." The adding interferometer first adds the signals from each receiver together and then squares that signal, which is what is measured in this lab. A single receiver would detect the signal from both sources simultaneously. **Let's think about what such a signal would look like.**

1.4 Galactic rotation curve lab

In this lab, you tried to measure the rotation curve of the galaxy by observing the tracer molecule CO, which is a proxy for H_2. Mapping the structure of our galaxy is a difficult task in general because we must do it from within the galaxy itself. You again employed the hot-cold load method to measure the noise temperature of the receiver and did a tipping scan to measure the optical depth of water in the atmosphere. Review the geometry of the galaxy and the coordinates ℓ, b that we use. Use measured a brightness temperature, which corresponds to some amount of gas, in many velocity bins; the computer picked out the maximum velocity peak for you.

2 Kirchhoff diffraction theory

Suppose $E(x, y)$ is the electric field in the sky and $\mathcal{E}(x', y')$ is the electric field on the dish. Let $\mathrm{d}A$ be a differential "patch" on the dish. Let R be the distance from the origin to the observed point (x, y) and let r be the distance from the patch $\mathrm{d}A$ to the observed point. Let D be some characteristic size of the dish and define an angular coordinate system (θ, ϕ) from the origin, with ϕ in the x'–y' plane, and let θ' be an angle with respect to the vertical from the patch $\mathrm{d}A$.

We assume that the electric field on the dish can be written

$$\mathcal{E}(x', y', t') = \mathcal{E}(x', y')\, e^{i\omega t'}. \tag{1}$$

That is, the time component is completely separate from the spatial component and takes the form of sines and cosines. In general, we have

$$E(\theta, \phi) = -\frac{ik}{4\pi} \int \mathcal{E}(x', y') \left(\frac{e^{ikr}}{r}\right) (1 + \cos\theta')\, \mathrm{d}x'\, \mathrm{d}y'. \tag{2}$$

We also have, just from Cartesian geometry,

$$r^2 = (x - x')^2 + (y - y')^2 + z^2 \tag{3}$$

and

$$R^2 = x^2 + y^2 + z^2. \tag{4}$$

In the far field,

$$r \approx R\left(1 - \frac{xx' + yy'}{R} + \cdots\right) \tag{5}$$

and

$$\frac{e^{ikr}}{r} \sim \frac{e^{ikr}}{R}. \tag{6}$$

Moving to a non-dimensional coordinate system, where $\alpha = x/R$ and $\beta = y/R$,

$$E(\alpha, \beta) = -\frac{ik}{4\pi}\frac{e^{ikR}}{R}\int \mathcal{E}(x', y')\, e^{-2\pi i(x'\alpha + y'\beta)/\lambda}\, \mathrm{d}x'\, \mathrm{d}y' \tag{7}$$

where we have substituted the approximation for r into the exponential. What does this actually mean? **The Fourier transform of the electric field in the sky is the electric field on the dish.**

3 21 cm cosmology

The 21 cmline arises from the phenomenon where the spin vector of the electron in a neutral hydrogen atom "flips," becoming either aligned or anti-aligned with the spin vector of the proton. The figures below are taken from a nice review of 21 cmcosmology on the *arXiv*; you can read more about it there. I will assume that you are comfortable with the concept of redshift z. At about $z \sim 10^3$, we have the "surface of last scattering." From the Caltech NED glossary,

> According to the standard Big Bang theory, the early Universe was sufficiently hot for all the matter in it to be fully ionised. Under these conditions, electromagnetic radiation was scattered very efficiently by matter, and this scattering kept the Universe in a state of thermal equilibrium. Eventually the Universe cooled to a temperature at which electrons could begin to recombine into atoms, and this had the effect of lowering the rate of scattering. This happened at what is called the recombination era of the thermal history of the Universe. At some point, when recombination was virtually complete, photons ceased to scatter at all and began to propagate freely through the Universe, suffering only the effects of the cosmological redshift. These photons reach present-day observers as the cosmic microwave background radiation (CMB). This radiation appears to come from a spherical surface around the observer such that the radius of the shell is the distance each photon has travelled since it was last scattered at the epoch of recombination. This surface is what is called the last scattering surface.

The first star formed at about $z \sim 50$, but this is *not* Cosmic Dawn. Cosmic Dawn occurs at about $z \sim 30$, when there are "knots" of stars. At about $z \sim 17$ to $z \sim 10$, we have the epoch of reionization.

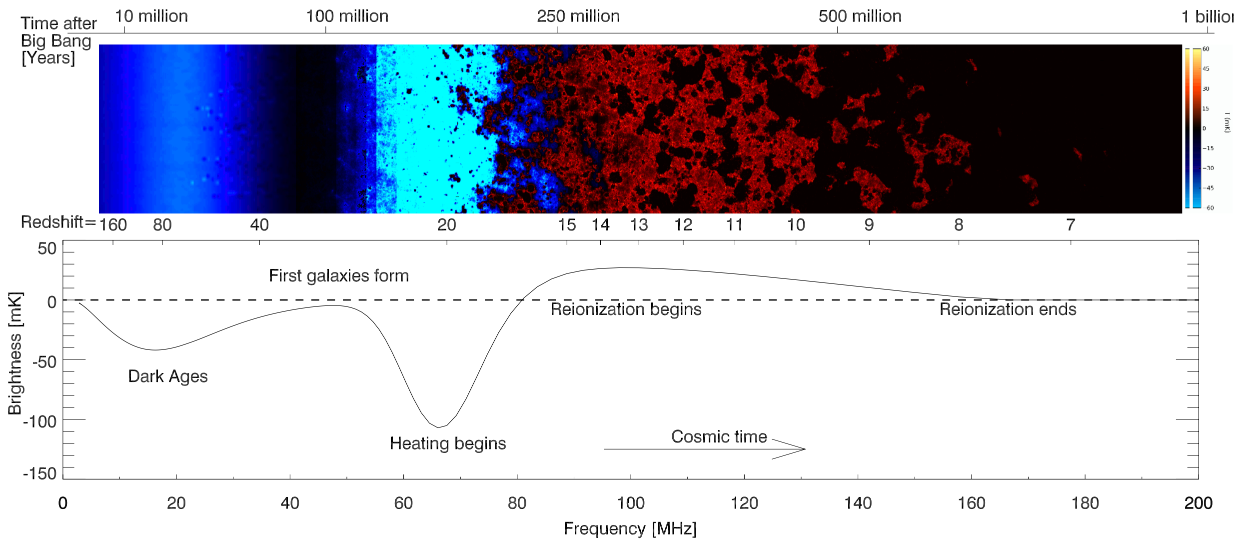

Figure 1: *Image credit Pritchard & Loeb (2012), arXiv:1109.6012.*

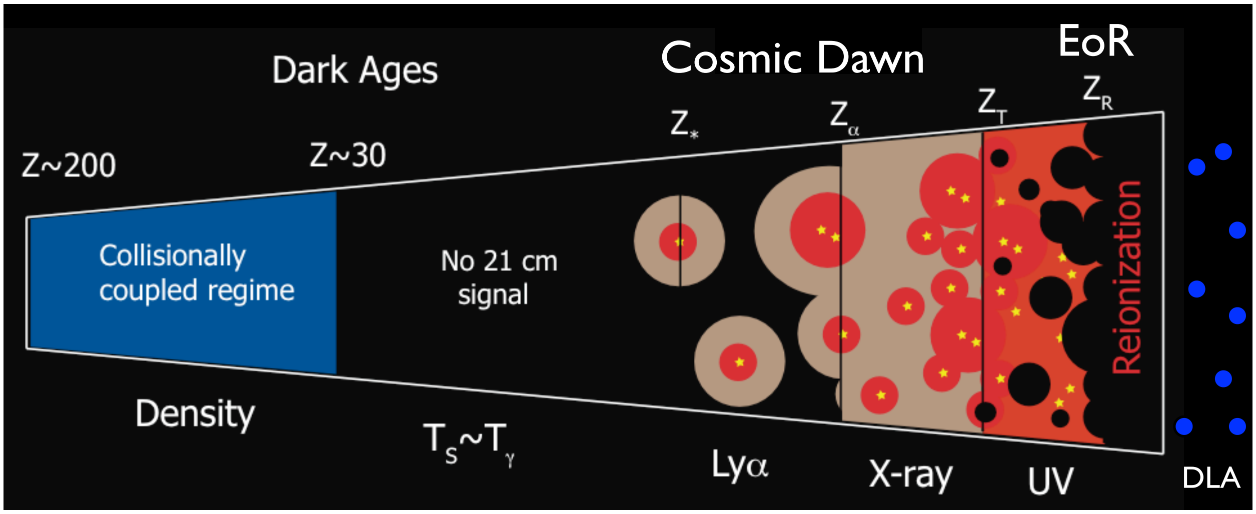

Figure 2: *Image credit Pritchard & Loeb (2012), arXiv:1109.6012.*